

LOW-BETA INSERTIONS INDUCING CHROMATIC ABERRATIONS IN STORAGE RINGS AND THEIR LOCAL AND GLOBAL CORRECTION

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Abstract

The chromatic aberrations induced by low- β insertions can seriously limit the performance of circular colliders. The impact is twofold: (1) a substantial off-momentum beta-beating wave traveling around the ring and leading to a net reduction of the mechanical aperture of the low-beta quadrupoles but also impacting on the hierarchy of the collimator and protection devices of the machine, (2) a huge non-linear chromaticity which, when combined with the magnetic imperfections of the machine, could substantially reduce the momentum acceptance of the ring by pushing slightly off-momentum particles towards non-linear resonances. These effects will be analyzed and illustrated in the framework of the LHC insertions upgrade Phase I [1] and a strategy for correction will be developed, requiring a deep modification of the LHC overall optics.

INTRODUCTION AND MOTIVATION

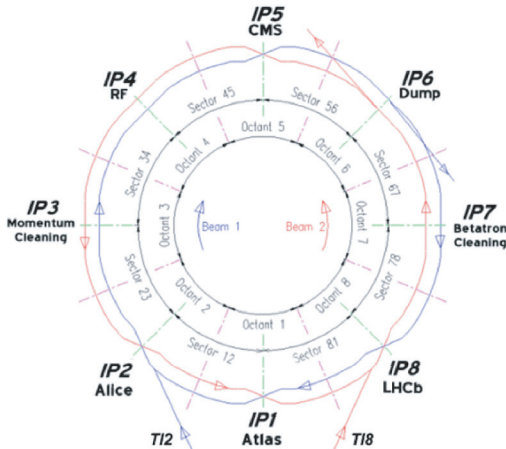


Figure 1: Schematic layout of the LHC.

The compensation of the chromatic aberrations arising from low- β insertions of a large storage ring is an old problem for which two types of solution are generally proposed.

- **Option I:** the adjustment of the betatron phase advances between two consecutive interaction points (IP) [2].
- **Option II:** the use of more than one sextupole family per plane (implemented in the LHC as of Version V4.1).

Option I. The first strategy assumes several layout and operation related constraints. First, the ring shall host an even number of low- β insertions (IR) and two consecutive IR's must be identical in terms of the following quantities:

$$I_{x,y}^{L,R} \stackrel{\text{def}}{=} \pm \int_{\text{Triplet}(L,R)} ds K_1(s) \beta_{x,y}(s), \quad (1)$$

which, for a given insertion and within a factor 4π , represent the contributions of the left and right low-beta

quadrupoles (inner triplet for the LHC) to the natural chromaticity of the ring. In practice, this strategy assumes the same layout, but also the same β^* and a perfect control of the linear optics in the two insertions. In particular one experiment could not run at full luminosity if the other can not for whatever reason. Then, in the specific case of two strictly identical low- β insertions, e.g. IR1 and IR5 for the LHC (see Fig. 1), the first chromatic derivative of the β functions and the second order chromaticity are given by the following expressions [2]:

$$\begin{cases} \frac{(\partial \beta / \partial \delta)_{(\delta=0;s)}}{\beta(s)} = -\frac{I^R + I^L}{\sin(2\pi Q)} \cos[\mu^{15} - 2\mu(s)] \cos[\mu^{15} - 2\pi Q] \\ \frac{(\partial^2 \beta / \partial \delta^2)_{(\delta=0;s)}}{\beta(s)} = -\frac{I^R + I^L}{\sin(2\pi Q)} \cos[\mu^{15} - 2\mu(s) + 2\pi Q] \cos[\mu^{15}], \end{cases} \quad (2)$$

between IP1 and 5, and between IP5 and 1, resp., and

$$Q'' = -\frac{(I^R + I^L)^2}{2\pi \sin(2\pi Q)} \cos(\mu^{15}) \cos(\mu^{15} - 2\pi Q), \quad (3)$$

where, in a given plane, Q denotes the betatron tune and μ^{15} the phase advance from IP1 to IP5. Therefore Q'' can be canceled via an appropriate phasing of the two interaction points (compare Figs. 2(a) and 2(c), with $\beta^* = 25$ cm in IP1 and 5 and a non-squeezed optics in IR2 and 8):

$$\mu_{x,y}^{15} = \frac{\pi}{2} \bmod[\pi] \text{ or } \mu_{x,y}^{15} = 2\pi Q_{x,y} + \frac{\pi}{2} \bmod[\pi]. \quad (4)$$

In this case, however, the off-momentum β -beating, to first order in δ_p , is canceled in half of the ring but is maximized in the other half (see Fig. 2(d)). Therefore, this strategy does not allow to compensate the effect simultaneously in the two LHC collimation insertions IR3 and IR7. This then may spoil the hierarchy of some LHC collimation and protection devices, in particular with primary collimators becoming secondary and conversely for momentum offsets δ_p as small as a few 10^{-4} . Finally, the condition (4) itself also depends strongly on δ_p (for a non-local correction of Q'). In other words, a substantial 3rd order chromaticity Q''' is expected (see Fig. 2(c)) together with a 2nd or higher order off-momentum β -beating even in the part of the ring where the 1st order has been corrected (see Figs. 2(e) and 2(f)).

Option II. As a back-up solution, the 8 LHC sectors were equipped with two interleaved sextupole families per plane and per beam, at least in view of an active correction of Q'' in collision. Neglecting the 2nd order dispersion, Q'' can indeed be expressed via the following integral [2]:

$$Q''_{x,y} \approx \mp \frac{1}{4\pi} \int_{\text{Ring}} ds [K_1(s) - K_2(s) D_x(s)] (\partial \beta_{x,y})_{(\delta=0;s)}, \quad (5)$$

with $K_{1,2}(s)$ denoting the strength of the quadrupoles and sextupoles of the lattice. As a result, noting that the off-momentum beta-beating oscillates at twice the betatron frequency (see Eq. (2)) and that the phase advance per cell is

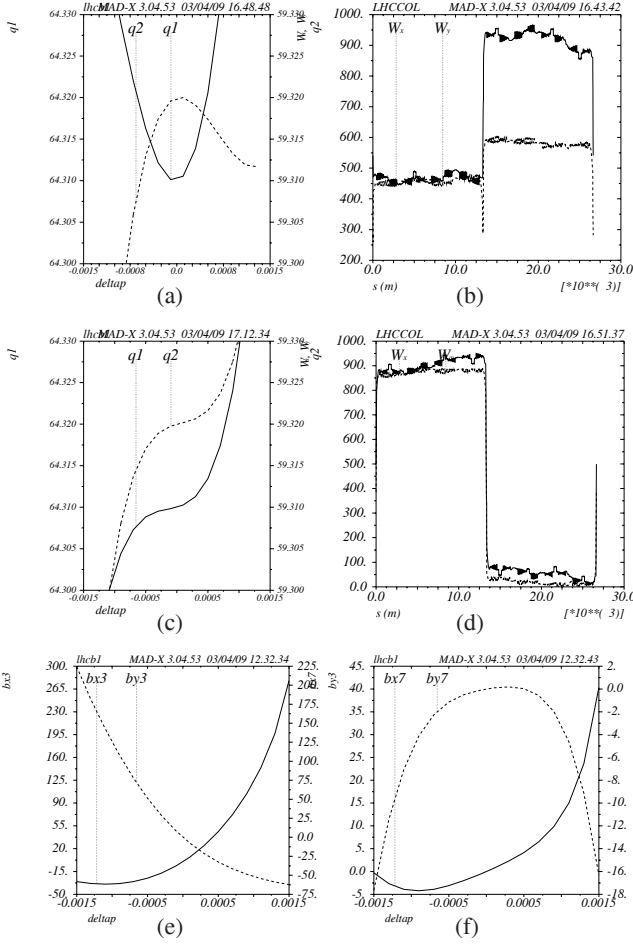


Figure 2: LHC Version 6.503 ($Q_{x,y} = 64.31/59.32$) with $\beta^* = 25$ cm in IP1 and 5, Q' matched to 2 units, before (top) and after (bottom) rephasing IP1 w.r.t. IP5: tunes vs δ_p (Fig.'s (a) and (c)), amplitude of the off-momentum β -beating (as defined in Eq. (2)) around the ring (Fig.'s (b) and (d)), and chromatic variations of the β -functions [%] in the collimation insertions IR3 and IR7 after IP phasing (Fig.'s (e) and (f)).

close to $\pi/2$ in the LHC arcs, an up and down powering strategy of the sextupole circuits (around a non-zero average value for Q' correction) offers a straightforward orthogonal knob to act on Q'' (see sketch in Fig. 3). This strategy, however, cannot warrant a magic correction of the off-momentum β -beating in the low- β quadrupoles and, if needed, as well as in the collimation insertions of the ring.

δ_P -FREE OPTICS

Phasing conditions. In order to reach this goal, dedicated constraints have to be imposed on the betatron phase advances all around the ring. In a given plane and a given side of the low- β insertion, the first set of constraints is obtained by requesting that the (2,0) and (0,2) chromatic driving terms induced by the triplet and the arc sextupoles (excited in up and down mode) compensate each other. The

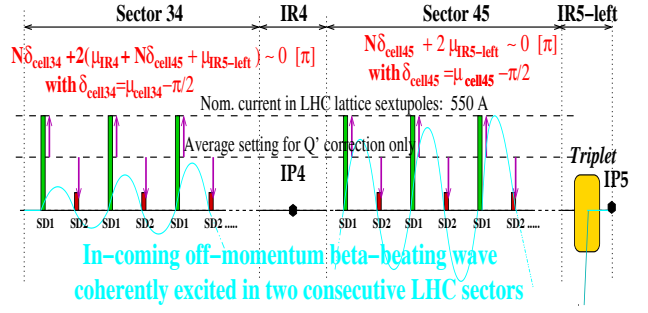


Figure 3: Phasing conditions w.r.t. IP5 and schematic powering scheme of the (SD) sextupole families in arcs 34 and 45 in order to excite coherently a (vertical) off-momentum β -beating wave.

phase advance of the triplet w.r.t. the IP being close to $\pi/2$ when the optics is squeezed, this condition, expressed hereafter for the left side of the IR, leads to

$$I_{x,y}^L \equiv e^{2i\phi_{x,y}^L} \sum_{k=0}^{N_c-1} (-1)^k \Delta K_2^{F,D} \beta_{x,y}^{F,D} D_x^{F,D} e^{2ik\mu_{x,y}^c} = \beta_{x,y}^{F,D} D_x^{F,D} \Delta K_2^{F,D} \frac{\sin[N_c \delta\mu_{x,y}^c]}{\sin[\delta\mu_{x,y}^c]} e^{i[N_c \delta\mu_{x,y}^c + 2\phi_{x,y}^L]}, \quad (6)$$

with $\beta_{x,y}^{F,D}$ and $D_x^{F,D}$ denoting the H (resp. V) β -function and the dispersion function at the focusing (defocusing) sextupoles SF (SD), $\Delta K_2^{F,D}$ the up and down excitation strength of the SF (SD) families, N_c the number of arc cells, $\delta\mu_{x,y}^c = \mu_{x,y}^c - \pi/2$ the deviation of the arc cell phase advance w.r.t. 90 degrees, and $\phi_{x,y}^L$ the H (V) betatron phase advance on the left side of the insertion, i.e. from the last SF (SD) sextupole up to the IP. The quantities $I_{x,y}^{L,R}$ being real and positive (see Eq. (1)), the above relation fixes the sign of ΔK_2 (i.e. determines which SD circuit plays the role of the SD1 family in Fig. (3)) and imposes

$$N_c \delta\mu_{x,y}^c + 2\phi_{x,y}^L \equiv 0 \mod [\pi], \quad (7)$$

idem for the right side of the IR. Then, in case more than one sector is needed for the chromatic correction of one triplet (which is the case for the LHC for $\beta^* \lesssim 50$ cm corresponding to $I_{x,y}^{L,R} \gtrsim 175$), a similar condition must be imposed on the second sector, constraining in particular the phase advance of the intermediate insertion (IR4 in Fig. 3) which plays the role of phase trombone w.r.t. to the $\partial_\delta \beta$ -wave generated in the two consecutive sectors:

$$\Delta\Phi_{x,y}^{IR} \stackrel{\text{def}}{=} N_c [\delta\mu_{x,y}^{c1} + \delta\mu_{x,y}^{c2}] + 2\phi_{x,y}^{IR} \equiv 0 \mod [\pi], \quad (8)$$

with $\phi_{x,y}^{IR}$ denoting the H (V) phase advance across the trombone insertion, more precisely between the first and last SF (SD) sextupole of the first and second sector. Finally, in the H (resp. V) plane, it is clear that the SD (SF) families also contribute to the H (V) off-momentum beta-beating wave. For the LHC these crossed contributions can substantially degrade the quality of the correction, corresponding to about 25% of the main contribution and being

in quadrature of phase with respect to it (for $\mu_{x,y}^c \sim \pi/2$). A second constraint shall then be imposed on the phase advances of the trombone insertions such that the cross-terms induced by two consecutive sectors compensate each other:

$$\Delta\Phi_y^{IR} - \Delta\Phi_x^{IR} = \pi \mod [2\pi]. \quad (9)$$

Correction efficiency. Coming back to the decoherence term (sine term) occurring in Eq. (6), the sextupole efficiency is obviously optimum when the phase advance per arc cell is equal to $\pi/2$ in both planes. By convention, the LHC IR's are defined from Q13 to Q13, where Q13.L/R are defocusing/focusing for the clock-wise rotating beam (B1) in IR1 and IR5, conversely for the trombone IR's, and conversely for the other beam (B2). Using this convention and the above phasing conditions, an optimum choice for the phases across the various LHC IR's would then be:

$$\begin{aligned} \phi_x^{L,B1} &\equiv \frac{\pi}{4} \mod \left[\frac{\pi}{2}\right] \text{ and } \phi_x^{R,B1} \equiv 0 \mod \left[\frac{\pi}{2}\right] \\ \phi_y^{R,B1} &\equiv \frac{\pi}{4} \mod \left[\frac{\pi}{2}\right] \text{ and } \phi_y^{L,B1} \equiv 0 \mod \left[\frac{\pi}{2}\right], \end{aligned} \quad (10)$$

concerning the left and right phase advances of IR1 and IR5 for B1 (conversely for B2), and

$$\phi_{x,y}^{IR} \equiv \frac{\pi}{4} \mod \left[\frac{\pi}{2}\right] \text{ and } \phi_x^{IR} - \phi_y^{IR} \equiv \frac{\pi}{2} \mod [\pi] \quad (11)$$

for the overall phase of the trombone IR's 2, 4, 6 and 8.

Quality of the correction. These results were not available during the design phase of the nominal LHC. Then, due to the limited tunability of the LHC IR's (either for mechanical aperture or quadrupole strength related reasons at injection or in collision), the above optimum IR phases can actually not be met. As a result, the phase advances of the LHC arc cells must be detuned w.r.t. $\pi/2$ in order to satisfy the phasing conditions (7), (8) and (9), at least in average for the two beams. Indeed, another LHC specific feature is that the main arc quadrupoles cannot act individually on both beams. Only the tune shift quads MQT's (equipping Q14 up to Q22) can do so and must now be used to overcome the limited tunability of the LHC IR's. A clear signature is a residual Q'' after correction, no longer due to the inner triplets in which the incoming $\partial_\delta\beta$ -wave is vanishing (see Eq. (5)) but induced by the sextupoles themselves (for which $\partial_\delta\beta(s) \neq 0$ when $\mu_{x,y}^c \neq \pi/2$).

A new overall LHC optics has designed accordingly in order to warrant a proper correction of the huge chromatic aberrations which will be generated in collision after the Phase I upgrade of the LHC inner triplets. This new optics, SLHCV1, corresponds to a reduced tune split of 3 ($Q_{x,y} = 63.31/60.32$). Due to strength limitations in the SD sextupole families (twice less efficient than the SF's with a dispersion reduced by a factor of ~ 2 at the QD's), β^* cannot be squeezed below 30 cm for Phase-I. The results obtained are illustrated in Fig. 4, in particular in terms of non-linear chromaticity (before and after fine tuning, if needed, of the residual Q'' by the Landau octupoles of the ring) or in terms of chromatic variations of the β -functions at strategic locations which are the LHC inner triplets of IR1 and IR5 and the collimation insertions IR3 and IR7.

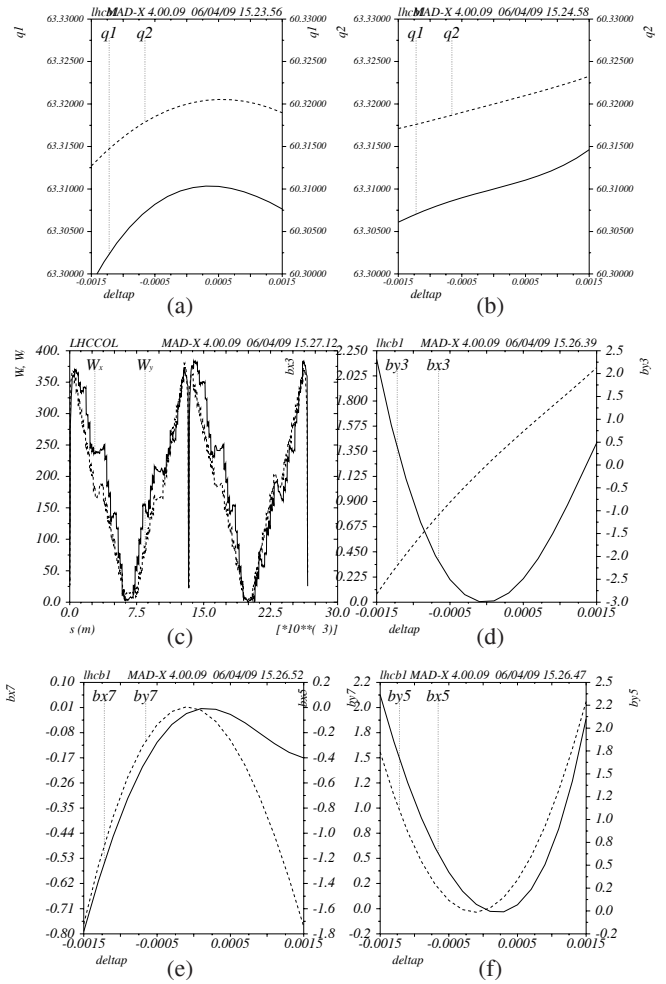


Figure 4: New optics SLHCV1 ($Q_{x,y} = 63.31/60.32$, $\beta^* = 30$ cm in IP1 and 5) after off-momentum β -beat correction (Q' matched to 2 units): tunes vs δp with and w/o fine tuning of Q'' via the arc octupoles MO (top), $\partial_\delta\beta$ -wave amplitude around the ring (Fig.(c)), and chromatic variations of the β -functions [%] in the collimation IR's IR3 and IR7 and in the IT of IR5 (Fig.'s (d)-(f)).

SUMMARY AND OUTLOOK

Low- β insertions can induce huge chromatic aberrations which, in the case of the LHC Upgrade Phase-I, constitute a severe obstacle for the viability of the project. A deep modification of the overall LHC optics is then mandatory to solve the problem. A multitude of checks is nevertheless required to fully validate the new optics proposed, in particular in terms of dynamic aperture at injection with phase advances going back towards ~ 90 degrees in the arc cells, as it was the case in the early optics versions of the LHC.

REFERENCES

- [1] *Conceptual Design of the LHC Interaction Region Upgrade: Phase-I*. LHC Project Report 1163, Nov. 2008.
- [2] S. Fartoukh. *Second order chromaticity correction of LHC V6.0 at collision*. LHC Project Report 308, Oct. 1999.